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(Un)attractor black holes in higher derivative AdS gravity

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Abstract: We investigate five-dimensional static (non-)extremal black hole solutions in higher derivative Anti-de Sitter gravity theories with neutral scalars non-minimally coupled to gauge fields. We explicitly identify the boundary counterterms to regularize the gravitational action and the stress tensor. We illustrate these results by applying the method of holographic renormalization to computing thermodynamical properties in several concrete examples. We also construct numerical extremal black hole solutions and discuss the attractor mechanism by using the entropy function formalism.

Keywords: Black Holes in String Theory, AdS-CFT Correspondence.

Contents

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1. Introduction

Among the fundamental interactions, gravity is very special. Gravity couples via a dimensional coupling constant, the Newton constant G_N, and so it is intrinsically non-renormalizable. Irrespective of the fundamental nature of quantum gravity, the gravitational low-energy degrees of freedom are encoded in the metric of spacetime itself. However, there is no reason to believe that the effects of our present theory are the whole story at the highest energies. Indeed, non-renormalizability can be interpreted as a natural feature of a theory for which the action is not fundamental but arises as an effective action in some energy limit. At high enough energies — for sufficiently strong curvatures and sufficiently small distances — new interactions and new degrees of freedom will be required.

The fact that the gravitational action is proportional to R *and only* R is not due to any symmetry and, unlike other theories, can not be argued on the basis of renormalizability. Indeed, the low energy effective gravity action that obeys principle of equivalence and general covariance has a generic structure. That is the usual Einstein action plus a series

of all possible interactions which are consistent with general covariance and local Lorentz invariance, i.e., higher curvature terms and also higher derivative terms involving the 'low-energy' matter fields.

The effects of heavy particles appear to be local interactions when viewed at low energy. That is the fields at different spacetime points are independent degrees of freedom with independent quantum fluctuations. One important caveat related to the interpretation of gravity as a (local) effective field theory is as follows: in a local field theory one expects an entropy proportional to the volume, but that is not true for black holes. In classical gravity, a fixed energy-density in a sufficiently large volume will collapse into a black hole.[1] However, the holographic principle [1] was proposed to rescue this situation: gravity in D dimensions is equivalent with a local field theory in $D-1$ dimensions. The AdS/CFT correspondence [2] (see [3] for a nice recent set of lectures) is a concrete realization of the holographic principle. Such correspondence is referred to as duality in the sense that the supergravity (closed string) description of D-branes and the field theory (open string) description are different formulations of the same physics. This way, the infrared (IR) divergences of quantum gravity in the bulk are equivalent to ultraviolet (UV) divergences of dual field theory living on the boundary. When we specify the CFT and say on which space it lives we are implicitly providing a set of counterterms for the gravity solution. These counterterms are local and depend only on the intrinsic boundary geometry [4, 5] (see, also, the reviews [6]) — a different method was proposed in [7]. Thus, one can compute the thermodynamical quantities in the gravitational side by employing the quasilocal formalism of Brown and York [8] supplemented by the boundary counterterms. The connection between the holographic charges and the various alternative definitions of conserved charges in AdS was explored in [9].

In studying string theories at low energy scales, the massive states may be integrated out to yield an effective action for the massless modes, with the same symmetries as the original string theory. Thus while the (super)gravity action is unique if we restrict to terms with two derivatives, interactions quadratic or higher order in the curvature tensor are allowed by the symmetries and so appear as well. However, such terms will require a *dimensional constant* to appear along with the derivatives. In string theory this constant turns out to be α', the inverse string tension.[2]

In this paper we investigate charged AdS black holes in the presence of higher derivative terms. We must note that, unlike in general relativity, in the presence of higher derivative corrections there are two families of solutions. We propose counterterms that regularize the action and the stress tensor of both branches (for horizons with spherical, toroidal, and hyperbolic topologies).

We obtain the stress tensor and the conserved charges for *exact* static charged non-extremal black hole solutions with Gauss-Bonnet (GB) term and find perfect agreement

[1] In quantum gravity, the existence of local operators is problematic due to the causality. It is well known that the commutator of space-like separated local operators should be zero. However, since the gravity is *dynamical* the metric itself fluctuates and so the space-like intervals are not well defined.

[2] This constant defines what is meant by 'slowly varying fields' in the sense that the derivative corrections may be ignored for fields that are slowly varying on the scale of the string length $l_s \sim \sqrt{\alpha'}$.

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with Wald formalism [10]. In the extremal limit we explicitly show that the near horizon geometry of the solution remains $AdS_2 \times S^3$ after including α'-corrections. The results we obtain provide a robust check of the entropy function formalism [11]. Indeed, we find that, for our *exact* solutions, the radius of AdS_2 receives corrections but the near horizon geometry remains $AdS_2 \times S^3(H^3)$. In this way we obtain the generalization of Bertotti-Robinson geometries [12] with GB term.

We also apply the counterterm method to 5-dimensional charged black hole solutions in gravity theories with U(1) gauge fields and neutral scalars. We obtain numerical solutions and generalize the results of [13] by including the higher derivative terms.

In the extremal limit we study the attractor mechanism by using the entropy function formalism [11, 14]. This method is based on the near horizon geometry and its enhanced symmetries but does not provide a proof for the existence of a complete solution in the bulk. For some special values of the couplings, we present numerical solutions with a finite horizon — this confirms the results in [15] where the equations of motion in the bulk were solved perturbatively order by order. Thus, we can safely apply the entropy function formalism.

An overview of the paper is as follows: in section 2 we study in detail the AdS charged black holes with GB term. We compute the stress tensor and the conserved charges of the exact non-extremal black hole solution by using the counterterm method and compare with the results obtained by Wald formalism. We present a preliminary discussion on the thermodynamics in both, canonical and grand-canonical ensembles. We also study the extremal limit and interpret our results within the entropy function formalism. Section 3 is dedicated to studying black hole solutions in AdS gravity with U(1) gauge fields non-minimally coupled to scalars in the presence of GB term. We present numerical non-extremal solutions and discuss in detail their properties by using the counterterms proposed in section 2. In section 4 we study the extremal limit in the case of massless scalar fields, construct numerical solutions, and investigate the attractor mechanism for these solutions. In section 5, we discuss our results. An appendix gives some calculational details on Wald formalism.

2. Charged AdS black holes with Gauss-Bonnet term

In this section we compute the conserved charges of AdS charged black holes with GB term by using both, the counterterm method and Wald formalism. We postulate counterterms that regularize the action and the stress tensor. The GB term is a very natural correction term to the Einstein action in the sense that the equations of motion contain no more than second derivatives in time. The main reason we are interested in GB term is due to the existence of *exact* solutions. In the extremal limit, we explictly check that the near horizon geometry still remains $AdS_2 \times S^3(H^3)$. We also use the entropy function formalism to interpret our results.

2.1 Non-extremal case

As we are interested in AdS gravity with higher derivatives (see [16] for a recent review), we

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begin by establishing our conventions for the action. Exact solutions are presented in [17]. Note, however, that our conventions differ from the ones in [17] and we also correct some important typos.

In this section, we will focus on a 5-dimensional theory of gravity with negative cosmological constant coupled to vector field, whose general action has the form

$$I = -\frac{1}{K_5^2}\int_M d^5x\sqrt{-g}[R - 2\Lambda - F_{\mu\nu}F^{\mu\nu} + \alpha' L_{GB}] \tag{2.1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the gauge field, Λ is the cosmological constant, and $K_5^2 = 16\pi G$. We use Gaussian units so that factors of 4π in the gauge fields can be avoided. The GB term $L_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$ appears in the low-energy effective *bosonic* string theory — in type IIB superstring the leading corrections are cubic in α'.

Within this theory there is a straightforward generalization of the Reissner-Nordstrom (RN) solution

$$ds^2 = -N(r)dt^2 + N(r)^{-1}dr^2 + r^2 d\Sigma_3^2 \tag{2.2}$$

with

$$N(r) = k + \frac{r^2}{4\alpha'}\left[1 + \epsilon\sqrt{1 + 8\alpha'\left(\frac{m}{r^4} - \frac{1}{L^2} - \frac{q^2}{r^6}\right)}\right], \quad A_\mu = \left(-\frac{\sqrt{3}q}{r^2} + \Phi\right)\delta_{\mu t} \tag{2.3}$$

where Φ is a constant which is chosen such that $A_t(r_h) = 0$ and r_h is the largest positive root of $N(r)$ that is typically associated to the outer horizon of a black hole — note that the condition $N'(r_h) > 0$ implies the existence of a minimal allowed value of r_h. Here, L is the radius of AdS spacetime and it is related to the cosmological constant by $\Lambda = -6/L^2$ and $k = 1, 0, -1$ corresponds to black holes with spherical, planar, and hyperbolic horizon topologies. The expression of $N(r)$ has an extra parameter $\epsilon = \pm 1$, that implies the existence of two branches of solutions.

Let us discuss now some known limits of the solutions (2.3) — more details can be find in [17]. The minus-branch solution reduces in the limit of $\alpha' \to 0$ to the RN solution of the Einstein-Maxwell-Λ system, *i.e.* $N(r) = k - m/r^2 + q^2/r^4$. On the other hand, N diverges for the $\epsilon = +1$ branch and so there is no smooth limit in this case, since $N(r) = r^2/(2\alpha') + k - m/r^2 + q^2/r^4 - r^2/L^2$ as $\alpha' \to 0$.

The background approached asymptotically by these solutions corresponds to an AdS_5 spacetime with an effective radius

$$L_{\text{eff}} = L\sqrt{\frac{1+\epsilon U}{2}}, \quad \text{where} \quad U = \sqrt{1 - \frac{8\alpha'}{L^2}} \tag{2.4}$$

This limit ($m = q = 0$) corresponds to AdS with higher derivatives. This effective radius of AdS with higher derivative corrections will play an important role in the subsection 2.1.1 where we will define the counterterms for the action and the stress-energy regularization. It is clear that by adding higher derivative corrections (even for small α') the theory contains new solutions (in our case a new branch) unavailable in general relativity.

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2.1.1 The counterterm method

One way to compute the energy of a gravitational system is by enclosing it with a surface — the observers living on this surface can make measurements and compare the results. In the quasilocal formalism [8], the *surface* stress tensor for spacetime and matter is defined by

$$T^{ab} \equiv \frac{2}{\sqrt{\gamma}} \frac{\delta S_{cl}}{\delta \gamma_{ab}} \tag{2.5}$$

where γ_{ab} is the induced metric on the enclosing surface.

As usual in gravity theories, the action (2.1) should be supplemented with suitable boundary terms to obtain a well-defined variational principle. For Einstein gravity, one considers the Gibbons-Hawking surface term [18]

$$I_b^{(E)} = -\frac{1}{8\pi G} \int_{\partial \mathcal{M}} d^4x \sqrt{-\gamma} K \tag{2.6}$$

where $\gamma_{\mu\nu}$ and K are the induced metric and the trace of the extrinsic curvature of the boundary, respectively.

A similar term occurs for Gauss-Bonnet gravity and reads [19, 20]

$$I_b^{(GB)} = -\frac{1}{8\pi G} \int_{\partial \mathcal{M}} d^4x \sqrt{-\gamma} \left\{ 2\alpha' \left(J - 2E_{ab}^{(1)} K^{ab} \right) \right\} \tag{2.7}$$

where $E_{ab}^{(1)}$ is the four-dimensional Einstein tensor of the metric γ_{ab} and J is the trace of

$$J_{ab} = \frac{1}{3}(2KK_{ac}K_b^c + K_{cd}K^{cd}K_{ab} - 2K_{ac}K^{cd}K_{db} - K^2K_{ab}) \tag{2.8}$$

Variation of the action $I + I_b^{(E)} + I_b^{(GB)}$ now gives an expression that does not contain normal derivatives of δg_{ab}.

It is well known that the total action contains divergences even at tree-level — they arise from integrating over the infinite volume of spacetime. We regularize the divergences by using the procedure proposed in [5]. This technique was inspired by the AdS/CFT duality and consists in adding suitable counterterms I_{ct} to the action of the theory in order to ensure its finiteness.

We have found that the action of the solutions in this paper can be regularized by the following counterterms

$$I_{\rm ct} = \frac{1}{8\pi G} \int_{\partial \mathcal{M}} d^4x \sqrt{-\gamma} \left(c_1 - \frac{c_2}{2} \mathsf{R} \right) \tag{2.9}$$

where R is the curvature scalar associated with the induced metric γ.

At this point it is important to emphasize that we postulate these counterterms and not derive them systematically as in [4, 6]. In fact, it is known that the counterterms postulated in [5] are not the most general and so they can be used just for a class of solutions (see [4, 6] for a detailed discussion). Therefore, the counterterms we proposed are not guaranteed to suffice for more general asymptotically AdS solutions of (2.1).

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The consistency of the procedure requires

$$c_1 = -\frac{1}{L_{\rm eff}}(2+\epsilon U) \qquad c_2 = \frac{L_{\rm eff}}{2}(2-\epsilon U) \tag{2.10}$$

This counterterm is general and can be used to regularize the action of both branches.[3] For solutions with a well defined Einstein gravity limit, one finds as $\alpha' \to 0$, that $c_1 \to -3/L + \alpha'/L^3 + O(\alpha')^2$, $c_2 \to L/2 + 3\alpha'/2L + O(\alpha')^2$ that match the results in [22, 23] (see, also, [24]).

Gravitational thermodynamics is then formulated via the Euclidean path integral, where one integrates over all metrics and matter fields between some given initial and final Euclidean hypersurfaces. Semiclassically the total action is evaluated from the classical solution to the field equations. The thermodynamical system has a constant temperature

$$T_H = \frac{1}{\beta} = \frac{N'(r_h)}{4\pi} \tag{2.11}$$

where β is the periodicity of the Euclidean time determined by requiring the Euclidean section be free of conical singularities.

To evaluate the action, one express the bulk action as a total derivative

$$\frac{1}{2}(R - 2\Lambda + \alpha' L_{GB} - F^2) = \frac{1}{r^3}\left(-\frac{1}{2}r^3\, N' - \frac{3\,q^2}{r^2} + 6\,r\,\alpha'\ \ (N-k)\,N'\right)' \tag{2.12}$$

where a prime for a metric function denotes a derivative with respect to the radial coordinate r. After adding the boundary terms, one finds that the Euclidean action is finite and contains two terms, $I = I^{as} + I^{eh}$. These two terms represent the contributions from the boundary and the event horizon and their expressions are

$$I^{as} = \frac{3V_k\beta}{16\pi G}m + I_0^{as} \qquad \text{with} \quad I_0^{as} = k^2\frac{3\beta L_{\rm eff}^2 V_k}{64\pi G}(3\epsilon U - 2) \tag{2.13}$$

$$I^{eh} = \frac{\beta V_k}{8\pi G}\left(\frac{1}{2}(r_h^2 + 12k\alpha')N'(r_h) + \frac{3q^2}{r_h^2}\right) \tag{2.14}$$

with V_k the area of the surface Σ_k. One can easily verify that the action computed according to a background subtraction coincides with the above expression up to the Casimir term I_0^{as} (the background choice in this case corresponds to a $q = 0$ vacuum EGB-AdS solution).

Varying the total action (that contains the boundary terms (2.6), (2.7), and (2.9)) with respect to the boundary metric h_{ab}, we compute the divergence-free boundary stress-tensor

$$T_{ab} = \frac{1}{8\pi G}\left(K_{ab} - K\gamma_{ab} + c_1\gamma_{ab} + c_2 \mathsf{G}_{ab} + \frac{\alpha}{2}(Q_{ab} - \frac{1}{3}Q\gamma_{ab})\right) \tag{2.15}$$

where

$$\begin{aligned} Q_{ab} = {} & 2KK_{ac}K_b^c - 2K_{ac}K^{cd}K_{db} + K_{ab}(K_{cd}K^{cd} - K^2) \\ & + 2K\mathsf{R}_{ab} + \mathsf{R}K_{ab} - 2K^{cd}\mathsf{R}_{cadb} - 4\mathsf{R}_{ac}K_b^c \end{aligned} \tag{2.16}$$

[3] A discussion of the counterterm method for GB gravity with cosmological constant also appears in the forth-coming paper [21].

with R_{abcd} and R_{ab} denoting the Riemann and Ricci tensors of the boundary metric.

Provided the boundary geometry has an isometry generated by a Killing vector ξ^i, a conserved charge

$$\mathfrak{Q}_\xi = \oint_\Sigma d^3S^i\ \xi^j T_{ij} \tag{2.17}$$

can be associated with a closed surface Σ [5]. Physically, this means that a collection of observers on the hypersurface whose metric is h_{ij} all observe the same value of $\mathfrak{Q}_\xi$ provided this surface has an isometry generated by ξ. The mass/energy M is the conserved charge associated with the Killing vector $\xi = \partial/\partial t$. For charged black holes, the expression of the nonvanishing components of the boundary stress tensor are

$$8\pi G T^w_w = \left(\frac{1}{2}mL_{\text{eff}} - k^2\frac{L^3_{\text{eff}}}{8}(2-3\epsilon U)\right)\frac{1}{r^4} + O(1/r^6) \tag{2.18}$$

$$8\pi G T^t_t = \left(-\frac{3}{2}mL_{\text{eff}} + 3k^2\frac{L^3_{\text{eff}}}{8}(2-3\epsilon U)\right)\frac{1}{r^4} + O(1/r^6) \tag{2.19}$$

where w denotes an angular direction on Σ_3 (note that in $1/r^4$ order, this is a traceless stress tensor).

Due to its high degree of symmetry, AdS space has a simple form in a large number of coordinate systems. By choosing different foliations of the spacetime one can describe boundaries that have different topologies and geometries (metrics), affording study of the CFT on different backgrounds. Specifically, we found additional Casimir-type contributions to the total energy depending on the slicing topology in accord with the expectations from quantum field theory in curved space. This can be seen for the solutions discussed in this section, whose mass computed according to (2.17) is

$$M = \frac{3V_k}{16\pi G}m + k^2\frac{3L^2_{\text{eff}}}{64\pi G}V_k(3\epsilon U - 2) \tag{2.20}$$

where the last term is the Casimir energy.

The metric on which the boundary CFT is defined is found by getting rid of the divergent conformal factor, $h_{ab} = \lim_{r\to\infty}\frac{L^2_{\text{eff}}}{r^2}\gamma_{ab}$, and corresponds to

$$h_{ab}dx^a dx^b = -dt^2 + L^2_{\text{eff}}d\Sigma^2_3 \tag{2.21}$$

If such a CFT exists the theory lies in the landscape of string theory and the bulk theory is manifestly consistent as an effective theory, otherwise the theory is part of the swampland [25].

2.1.2 Wald formalism

One way of understanding black hole entropy comes from the use of Euclidean analog of a black hole spacetime. Whenever it is not possible to foliate the Euclidean section of a given (stationary) spacetime by a family of surfaces of constant time, gravitational entropy will emerge. Another approach to gravitational entropy is the Noether charge formalism of Wald. The relation between the two methods was explored in [26] (see, also, [27, 28]).

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When we add R^2 corrections to the action the entropy is no longer given by the area law — instead, to computing the entropy of the black holes (2.3), we will use a more general formula proposed by Wald [10]:

$$S = -2\pi \int_{\mathcal{H}} d^3x \frac{\partial L}{\partial R_{abcd}} \epsilon_{ab}\epsilon_{cd} \tag{2.22}$$

where $\mathcal{H}$ is the bifurcate horizon and $\epsilon_{\mu\nu}$ is the binormal to the bifurcation surface. Interestingly enough, the entropy can still be expressed as a local functional evaluated at the (bifurcate) horizon. In this construction, the entropy was obtained from the Noether charge that is the integral of a 3-form associated with the diffeomorphism invariance of the theory. It is worth noticing that Wald formalism can be applied to *non-extremal* black hole solutions in generally covariant theories of gravity.

The most general formula for the entropy for a Lagrangean of the form

$$I = \int d^5x\sqrt{-g}\left[\frac{R}{16\pi G} - 2\Lambda + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu} + \gamma R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right] \tag{2.23}$$

is given by (see apendix)

$$S = \frac{1}{4G}\int_{\mathcal{H}} d^3x\sqrt{h}\left[1 + 2K_5\alpha R + K_5\beta(R - h^{ij}R_{ij}) + 2K_5\gamma(R - 2h^{ij}R_{ij} + h^{ij}h^{kl}R_{ikjl})\right] \tag{2.24}$$

We are interested in GB term for which the expression for entropy becomes,

$$S = \frac{1}{4G}\int_{\mathcal{H}} d^3x\sqrt{h}\left[1 + 2\alpha' h^{i,k}h^{j,l}R_{ijkl}\right] \tag{2.25}$$

where

$$\alpha = \frac{\alpha'}{K_5}, \quad \beta = -\frac{4\alpha'}{K_5}, \quad \gamma = \frac{\alpha'}{K_5} \tag{2.26}$$

for GB term and $K_5 = 16\pi G$. It is easy to show that

$$h^{ik}h^{jl}R_{ijkl} = \frac{6}{r_+^2} \tag{2.27}$$

and hence entropy becomes

$$S = \frac{V_k}{4G} r_h(r_h^2 + 12k\alpha') \tag{2.28}$$

that matches with (2.32) obtained by the counterterm method.

2.2 The grand canonical and canonical ensembles

The results above make possible a discussion of the thermodynamic properties of these charged black hole solutions. In a very basic sense, gravitational entropy can be regarded as arising from the Gibbs-Duhem relation applied to the path-integral formulation of quantum gravity, which in the semiclassical limit yields a relationship between gravitational entropy and other relevant thermodynamic quantities. In this approach, the expression of the entropy is

$$S = \beta(M - \mu_i \mathfrak{C}_i) - I \tag{2.29}$$

upon application of the Gibbs-Duhem relation to the partition function, with chemical potentials $\mathfrak{C}_i$ and conserved charges μ_i. For the situation in this work, $\mathfrak{C}$ corresponds to the electrostatic potential Φ, while μ is the electric charge Q, with

$$\Phi = \frac{3q}{r_h^2}, \quad Q = \frac{V_k}{8\pi G} 2\sqrt{3} q \tag{2.30}$$

To compute the entropy, it is convenient to express everything in terms of (r_h, q)

$$T_H = \frac{r_h}{2\pi L^2} \frac{2r_h^2/L^2 - q^2/r_h^4 + k}{r_h^2/L^2 + 4k\alpha'/L^2}, \quad m = \frac{q^2}{r_h^2} + k r_h^2 + \frac{r_h^4}{L^2} + 2\alpha' k^2 \tag{2.31}$$

the action being given by the sum of (2.13) and (2.14). In this way, one finds the following expression for the black hole entropy:

$$S = \frac{V_k}{4G} r_h (r_h^2 + 12 k\alpha') \tag{2.32}$$

One can easily verify that the first law of thermodynamics $dM = T_H dS + \Phi dQ$ also holds.

The corresponding equation of state (analogous to $f(p, V, T)$, for, say, a gas at pressure p and volume V) reads

$$T_H = \frac{1}{6\pi L^2} \sqrt{\frac{Q}{\Phi}} \frac{3Q + 2\Phi(3k - 4\Phi^2)L^2}{Q + 16 k\alpha' \Phi}. \tag{2.33}$$

A discussion of the corresponding thermodynamical properties can also be approached. In a grand canonical ensemble one finds the Gibbs free energy

$$\begin{aligned} W[T_H, \Phi] &= M - T_H S - Q\Phi = W_0 + W_1, \\ \text{where} \quad W_0 &= \frac{V_k}{8\pi G} \frac{3k^2}{16} \left(L(L + \sqrt{L^2 - 8\alpha'}) - 8\alpha' \right), \\ W_1 &= -\frac{V_k}{8\pi G} \frac{Q^2(3Q + 4\Phi L^2(4\Phi^2 - 3k)) + 48\alpha' k \Phi Q (9Q + 4\Phi(3k - 4\Phi^2)L^2)}{96 L^2 \Phi^2 (Q + 16 k\alpha' \Phi)} \end{aligned}$$

where Q is given as $Q(T_H, \Phi)$ by the equation of state (2.33).

One can consider insted a canonical ensemble, where the temperature and electric charge are keept fixed. The Helmholtz potential $F = M - TS$ in this case is

$$\begin{aligned} F[T_H, Q] &= F_0 + F_1, \\ F_0 &= \frac{V_k}{8\pi G} \frac{3k^2}{16} \left(L(L + \sqrt{L^2 - 8\alpha'}) - 8\alpha' \right), \\ F_1 &= \frac{V_k}{8\pi G} \frac{Q^2(-3Q + 4\Phi(3k + 20\Phi^2)L^2 - 144\alpha' k \Phi Q (3Q + 4\Phi(k - 4\Phi^2)L^2)}{96 L^2 \Phi^2 (Q + 16 k\alpha' \Phi)} \end{aligned} \tag{2.34}$$

where the electrostatic potential Φ is given as a function of T_H, Q by the equation of state (2.33).

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2.3 Extremal case

2.3.1 Exact solution

The non-extremal black holes have a non-zero temperature that can be evaluated by eliminating the conical singularity in the Euclidean section. Once we impose the periodicity condition, the Euclidean time circle closes off smoothly and the Euclidean geometry becomes a 'cigar'. On the other hand, an extremal Euclidean black hole has a different topology. That is an infinite long throat for which the Euclidean time circle does not close off. In this case, one is forced to work with an arbitrary periodicity of the Euclidean time leading to ambiguos results (though, see [29]).

However, on the Lorentzian section the picture is quite satisfactory: an extremal black hole is obtained by continuosly sending the temperature of a non-extremal black hole to zero. While the temperature vanishes, the area of the horizon can remain finite.

Thus, to obtain the extremal black hole solution we work on the Lorentzian section. The extremal limit can be equivalently obtained by imposing the constraint that the horizon is degenerate (i.e., $N(r)$ has a double root: $N(r_H) = N'(r_H) = 0$).

One can easily solve the equations system to obtain:

$$m = 2\left(k(k\alpha' + r_H^2) + \frac{3}{2}\frac{r_H^4}{L^2}\right) \tag{2.35}$$

$$q^2 = r_H^6\left(\frac{k}{r_H^2} + \frac{2}{L^2}\right) \tag{2.36}$$

where r_H is the horizon radius.

Using the method of [30] (see section 4.2) it is straightforward to show that the near horizon geometry of (2.3) is $AdS_2 \times S^3(H^3)$ and just the radius of AdS_2 receives corrections (we will compute and provide its concrete value in section 2.3.2).

These geometries are interesting in their own right and provide the generalizations of Bertotti-Robinson geometries with GB term. These solutions are the topological product of two manifolds of constant curvature. They are conformally flat and are supported by a flux through $S^3(H^3)$:

$$ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + R_k^2 d\Sigma_3^2, \quad \text{with} \quad F(r) = k + \frac{4r^2(3R_k^2 + kL^2)}{L^2(R_k^2 + 4k\alpha')} \tag{2.37}$$

and satisfy the equations of motion with a gauge field $A_t = r\sqrt{12/L^2 + 6k/R_k^2}$ — here R_k is a constant and so the size of Σ_3^2 is fixed. Since H^3 is not compact, the solution also exists for a vanishig electric potential in this case.

2.3.2 Entropy function

Wald's construction for the entropy can be used in any general coordinate invariant theory of gravity including those with higher derivative terms in the action. This method was extended by Sen to extremal black holes and it is reffered to as the entropy function

formalism [11].[4] A discussion on the entropy function formalism and the Euclidean section method can be found in [33].

In this section we use the entropy function formalism and compare the results with the ones in the previous subsection. The general metric of $AdS_2 \times S^3$ can be written as

$$ds^2 = v_1(-\rho^2 d\tau^2 + \frac{1}{\rho^2} d\rho^2) + v_2 d\Omega_3^2 \,. \tag{2.38}$$

The field strength ansatz is $F = e d\tau \wedge d\rho$ and, for this geometry, the GB term comes out to be GB$= -24/v_1 v_2$. Thus, the entropy function $F(v_1, v_2, e, Q)$ is given by

$$\begin{aligned} F(v_1, v_2, e, Q) &= 2\pi[Qe - f(v_1, v_2, e)] \,, \\ f(v_1, v_2, e) &= 2\pi^2 \left[-2v_2^{3/2} + 6v_1\sqrt{v_2} + 2\frac{v_2^{3/2}}{v_1} e^2 + v_1 v_2^{3/2} \left(\frac{12}{L^2}\right) - 24\alpha'\sqrt{v_2} \right] . \end{aligned} \tag{2.39}$$

The attractor equations are:

$$\frac{\partial F}{\partial v_1} = 0 \;\Rightarrow\; 6v_1^2 - 2v_2 e^2 + v_1^2 v_2 \left(\frac{12}{L^2}\right) = 0 \,, \tag{2.40}$$

$$\frac{\partial F}{\partial v_2} = 0 \;\Rightarrow\; -v_1 v_2 + v_1^2 + v_2 e^2 + \frac{v_1^2 v_2}{2} \left(\frac{12}{L^2}\right) - 4\alpha' v_1 = 0 \,, \tag{2.41}$$

$$\frac{\partial F}{\partial e} = 0 \;\Rightarrow\; Q = 8\pi^2 \frac{v_2^{3/2}}{v_1} e \,. \tag{2.42}$$

Let us now discuss in detail these equations. One important observation is that by adding the GB term to the action just the second attractor equation is modified. One can easily eliminate v_1 from the first equation by using the third one and so the value of v_2 does not change — we obtain the following relation between the electric charge and the horizon radius ($v_2 = r_H^2$):

$$\tilde{Q}^2 = \left(\frac{Q}{8\pi^2}\right)^2 = 3v_2^2 \left(1 + \frac{2v_2}{L^2}\right) \tag{2.43}$$

Using the conventions from the section (2.1) and the relation between the physical electric charge and the charge parameter $\tilde{Q} = \sqrt{3}q$ we can see that this relation matches (2.35). It is worth emphasizing that just the radius of AdS_2 receives corrections

$$v_1 = \frac{1}{4} \frac{4\alpha' + v_2}{1 + 3v_2/L^2} \tag{2.44}$$

and so the entropy of extremal black hole is

$$S_{extremal} = 8\pi^3 r_H^3 \left(1 + \frac{12\alpha'}{r_H^2}\right) \tag{2.45}$$

The entropy of extremal black hole has the same form as for the non-extremal black hole (2.32) ($V_k = 2\pi^2$ and $G_N = 1/16\pi$), though the radius of the horizon (r_H) is different.

[4] It is known that the near horizon geometry of stationary extremal black holes contains an AdS_2 space [31] (see, also, [32]). The entropy function is constructed, on an SO(2, 1) × SO(3) (for static black holes) or SO(2, 1) × U(1) (for stationary black holes) symmetric (near horizon) background, by taking the Legendre transform (with respect to the electric charges and angular momentum) of the reduced Lagrangian evaluated at the horizon.

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3. Charged black holes with scalar hair

In this section we generalize the results of [13] by including the GB term. We obtain numerical solutions[5] and discuss how the GB term affects their properties by using the counterterms proposed in the previous section.

3.1 The model

We consider the generalization of the RN black holes in a five-dimensional theory of gravity coupled to a set of scalars and vector fields, whose general action has the form

$$I[G_{\mu\nu},\phi^I,A^B_\mu] = -\frac{1}{16\pi G}\int_M d^5x\sqrt{-g}[R+\alpha' L_{GB} - G_{IJ}(\phi)\partial_\mu\phi^I\partial^\mu\phi^J \\ -f_{AB}(\phi)F^A_{\mu\nu}F^{B\,\mu\nu} - V(\phi)], \tag{3.1}$$

where $F^A_{\mu\nu}$ with $A=(0,\dots N)$ are the gauge fields, $\phi\equiv(\phi^I)$ with $(I=1,\cdots,n)$ are the scalar fields, $V(\phi^i)$ is the scalar fields potential.

The equations of motion for the metric, scalars, and the gauge fields are given by [17, 15]

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + g_{\mu\nu}V(\phi) + \alpha' H_{\mu\nu} = 8\pi G T^{\rm matter}_{\mu\nu} \tag{3.2}$$

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}G_{IJ}(\phi)\partial^\mu\phi^J) = \frac{1}{2}\Big(\frac{\partial f_{AB}(\phi)}{\partial\phi^I}F^A{}_{\mu\nu}F^{B\,\mu\nu} \\ +\frac{\partial G_{KM}(\phi)}{\partial\phi^I}\partial_\mu\phi^K\partial^\mu\phi^M + \frac{\partial V(\phi)}{\partial\phi^I}\Big) \tag{3.3}$$

$$\partial_\mu\left[\sqrt{-g}\left(f_{AB}(\phi)F^{B\,\mu\nu}\right)\right] = 0 \tag{3.4}$$

where $T^{\rm matter}_{\mu\nu}$ is the matter stress tensor and $H_{\mu\nu}$ is given by [17]

$$H_{\mu\nu} = 2(RR_{\mu\nu} - 2R_{\mu\alpha}R^\alpha_\nu - 2R^{\alpha\beta}R_{\mu\alpha\nu\beta} + R^{\alpha\beta\gamma}_\mu R_{\nu\alpha\beta\gamma}) - \frac{1}{2}g_{\mu\nu}L_{GB} \tag{3.5}$$

The Bianchi identities for the gauge fields are $F^A{}_{[\mu\nu;\lambda]}=0$.

We assume that the scalar fields approache asymptotically constant values, ϕ^I_∞, which corresponds to an extremum of the potential such that $dV/d\phi\big|_{\phi_\infty}=0$ and $V(\phi_\infty) = -12/L^2<0$, with the expansion

$$V(\phi) = V(\phi_\infty) + \frac{1}{2}\frac{\partial^2 V}{\partial\phi^I\partial\phi^J}\bigg|_{\phi=\phi_\infty}\phi^I\phi^J + \dots \tag{3.6}$$

the scalar field masses being set by $\partial^2V/\partial\phi^I\partial\phi^J\big|_{\phi_\infty}=\mu_{IJ}$.

Under these asumptions, the background of the theory is given by the solution

$$ds^2 = -\left(k+\frac{r^2}{L^2_{\rm eff}}\right)dt^2 + \frac{dr^2}{k+\frac{r^2}{L^2_{\rm eff}}} + r^2 d\Sigma^2_3 \tag{3.7}$$

[5] We thank Eugen Radu for advice in finding the numerical solutions and explaining to us the methods used in [34].

with $\phi = \phi_\infty$, $k = \pm 1, 0$ and the effective length scale is $L_{\rm eff} = L\sqrt{1+U/2}$, with $U = \sqrt{1-8\alpha'/L^2}$ as in the case of EGB-Λ theory.

Restricting to static solutions, we consider the metric ansatz

$$ds^2 = \frac{dr^2}{N(r)} + r^2 d\Sigma_3^2 - N(r)\sigma(r)^2 dt^2, \tag{3.8}$$

and a purely-electric abelian field ansatz $A^B = W^B(r)dt$, the scalar fields being also functions only of the radial coordinate r.

The Maxwell equation implies the existence of the first integrals

$$W'^A = f^{AB} q_B \frac{\sigma}{r^3} \tag{3.9}$$

where q_B are constants fixing the electric charges of solutions, $Q_B = q_B V_k/(2\pi G)$ and f^{AB} is the inverse of f_{AB}. The electric potentials are the integrals of F^B_{rt}, being fixed up to arbitrary constants Φ^B which are chosen such that A^B_t vanish on the event horizon.

It is more convenient to combine the equations of motion (see [15]) to obtain the following equivalent system of differential equations:

$$\begin{aligned}
&\frac{3}{4} r(rN' + 2N - 2k) - 3\alpha'(N-k)N' + \frac{1}{4} r^3 N G_{IJ}\phi'^I \phi'^J + \frac{1}{4} r^3 V(\phi) + \frac{1}{2} f^{AB} q_A q_B = 0 \\
&\frac{\sigma'}{\sigma} = \frac{1}{12} \frac{r^3 G_{IJ}\phi'^I \phi'^J}{r^2/4 - \alpha'(N-k)} \\
&\frac{2}{r^3\sigma}(N r^3 \sigma G_{IJ}\phi^{J'})' = N\phi^{K'}\phi^{S'} \frac{\partial G_{KS}}{\partial \phi^I} + 2\frac{\partial f_{AB}}{\partial \phi^I} f^{AC} f^{BD} \frac{q_C q_D}{r^6} + \frac{\partial V}{\partial \phi^I}
\end{aligned} \tag{3.10}$$

The first equation does not contain any second derivatives and is the Hamiltonian constraint. We notice that the equations of motion can also be derived from the one-dimensional reduced Lagrangean:

$$\begin{aligned}
L_{\rm red} = &-3r\sigma(rN' + 2N - 2k) + 12\alpha'(N-k)\sigma N' - r^3 \sigma N G_{IJ}\phi^{I'}\phi^{J'} \\
&- r^3\sigma V(\phi) - \frac{2\sigma}{r^3} f^{AB} q_A q_B
\end{aligned} \tag{3.11}$$

We are interested in black hole solutions approaching asymptotically the background (3.7), that suggests to use the following form of the metric function, $N(r)$:

$$N(r) = k - \frac{m(r)}{r^2} + \frac{r^2}{L^2_{\rm eff}} \tag{3.12}$$

The first equation in (3.10) implies that $m(r)$ satisfies the following equation

$$\frac{3}{4}\left(Um + L^2_{\rm eff}(U-1)\frac{m^2}{2r^4}\right)' = \frac{1}{4} r^3 N G_{IJ}\phi^{I'}\phi^{J'} + \frac{1}{4} r^3 (V(\phi) - V(\phi_\infty)) + \frac{1}{2r^3} f^{AB} q_A q_B$$

The computation of the action and the stress tensor of these configurations can be done by using a similar approach to the one discussed in section 2 and we will not present the details here (without α' corrections, see [13]). Similar to the case without scalars, the

volume term in the action (3.1) has a total derivative structure and so it can be expressed in terms of the difference of two surface integrals.

The divergencies associated with the asymptotic AdS structure of the solutions can be removed by supplementing (3.1) with the same boundary counterterms (2.9) as in section 2. After Wick rotating $t \to i\tau$ to the Euclidean section, we found that the action can be written in the usual 'quantum statistical' form

$$I = \beta(M - Q_A\Phi^A) - \frac{V_k}{16\pi G} r_h(r_h^2 + 12\alpha' k) \tag{3.13}$$

where β is the periodicity of the Euclidean time. Here, M is the mass of the black hole and we will present its values for some concrete examples in the next sections.

The value of β is arbitrary for soliton solutions (that exist in the absence of gauge fields, e.g. [35]) or extremal black holes. However, the regularity of the Euclideanized solutions as $r \to r_h$ imposes

$$\beta = \frac{1}{T_H} = \frac{4\pi}{N'(r_h)\sigma(r_h)} \tag{3.14}$$

for non-extremal black hole solutions.

The mass of these solutions can be computed within the quasilocal formalism by using the generic relation (2.17), where the boundary stress tensor is still given by (2.15). However, the situation is different for theories with massless scalar fields and in the presence of massive scalar fields — we shall discuss these cases separately.

The explicit construction of solutions requires specification of the functions G_{IJ} and f_{AB}. In what follows we consider a model with one single scalar (i.e. $G_{IJ} = 2\delta_{1I}\delta_{1J}$, $\phi^1 = \phi$) and two gauge fields with modulus dependent couplings of the form

$$f_{AB}(\phi) = \delta_{AB} e^{\alpha_B \phi} \tag{3.15}$$

Moreover, we shall restrict to solutions with a smooth Einstein gravity limit.

3.2 Unattractor solutions with a massless scalar field

In their simplest version, these solutions have a constant value of the scalar potential, $V(\phi) = 2\Lambda = -12/L^2$, which is the case considered here. The generic solutions have a non-degenerate horizon and are easier to study. Near the event horizon, they admit a power series expansion of the form (here we restrict to the first terms in the series)

$$\begin{aligned} N(r) &= f_1(r - r_h) + \dots, \\ \sigma(r) &= \sigma_h + \frac{2r_h^3 \phi'^2(r_h)\sigma_h}{3r_h^2 + 4\alpha' k}(r - r_h) + \dots, \\ \phi(r) &= \phi_h - \frac{1}{2r_h^6 f_1}(\alpha_1 e^{-\alpha_1\phi_h} q_1^2 + \alpha_2 e^{-\alpha_2\phi_h} q_2^2)(r - r_h) \end{aligned} \tag{3.16}$$

where

$$f_1 = -\frac{2(-6r_h^6/L^2 - 3kr_h^4 + e^{-\alpha_1\phi_h} q_1^2 + e^{-\alpha_2\phi_h} q_2^2)}{3r_h^3(r_h^2 + 4\alpha' k)} \tag{3.17}$$

The coefficients of all higher order terms in the expression of N, σ, ϕ are fixed by the two parameters ϕ_h, σ_h. One can easily see that the condition $f_1 > 0$ imposes the existence of a minimal value of r_h for given values of ϕ_h, q_1, q_2.

One can also construct an approximate solution at the boundary in terms of three free parameters ϕ_∞, Σ, and M_0

$$\begin{aligned} N(r) &= 1 - \frac{M_0}{r^2} + \frac{r^2}{L_{\text{eff}}^2} + \frac{e^{-\alpha_1\phi_\infty} q_1^2 + e^{-\alpha_2\phi_\infty} q_2^2}{3Ur^4} + \dots, \\ \phi(r) &= \phi_\infty + \frac{\Sigma}{r^4} + \dots, \\ \sigma(r) &= 1 - \frac{4\Sigma^2}{3Ur^8} + \dots \end{aligned} \tag{3.18}$$

The next leading term at the boundary corresponds to a normalizable mode and the black hole is a state in the boundary CFT.

By applying the quasilocal formalism discussed above, one finds the mass of these solutions

$$M = \frac{V_k}{8\pi G} M_0 + k^2 \frac{3L_{\text{eff}}^2}{64\pi G} V_k (3U - 2) \tag{3.19}$$

while the entropy of solutions has the same form as in the RN case, $S = \frac{V_k}{4G} r_h (r_h^2 + 12\alpha' k)$

We will explictly check by our numerical analysis that M, r_h, and ϕ_h depend of the asymptotic boundary data (ϕ_∞). This is in contrast with the extremal case (see section 4) where we obtain an attractor behaviour of the horizon.

Although an exact solution of the equations of motion (3.10) appears to be intractable, here we present arguments for the existence of non-trivial solutions which smoothly interpolate between the asymptotic expansions (3.16) and (3.18).

Starting from the event horizon expansion (3.16) we integrated the equations towards $r \to \infty$. The integration stops when the asymptotic limit (3.18) is reached with a reasonable accuracy. In this approach, the input parameters are $k, r_h, q_1, q_2, \alpha_1, \alpha_2, L$ and the value ϕ_h of the scalar field on the horizon. The equation for σ decouples from the rest, and the requirement that $\sigma \to 1$ as $r \to \infty$ can be relaxed during the numerical integration, $\sigma(r_h)$ subsequently being multiplied by an appropriate constant factor so that the correct asymptotic behaviour is recovered.

We have solved the equations of motion for several values of $\alpha_1 = -\alpha_2 = 2a$ and a large set of r_h, q_1, q_2, ϕ_h, L. We follow the usual approach and, by using a standard ordinary differential equation solver, we evaluate the initial conditions (3.16) at $r = r_h + 10^{-4}$ for global tolerance 10^{-12}, for a fixed parameter ϕ_h and integrating towards $r \to \infty$.

The complete classification of the solutions in the space of parameters is a considerable task that is not aimed in this paper. Also, we shall restrict to the case of spherical topology horizon, although we have found topological black holes as well.

For all configurations we have studied, the metric functions $N(r)$, $\sigma(r)$, and the scalar $\phi(r)$ interpolate monotonically between the corresponding values at $r = r_h$ and the asymptotic values at infinity, without presenting any local extrema. In figure 1 we plot the profiles

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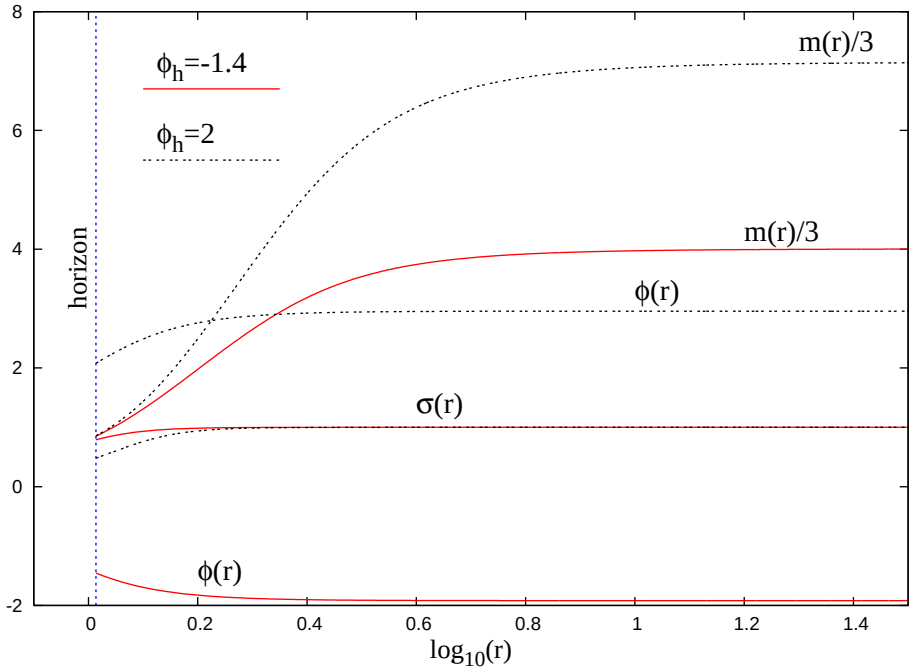

Figure 1: The profiles of the functions $m(r)$, $\sigma(r)$ and $\phi(r)$ are shown for typical $k = 1$ non-extremal black holes with with $r_h = 1, q_1 = 0.3, q_2 = 0.5, \Lambda = -6$, $\alpha' = 0.1$ and two values of the scalar field on the event horizon

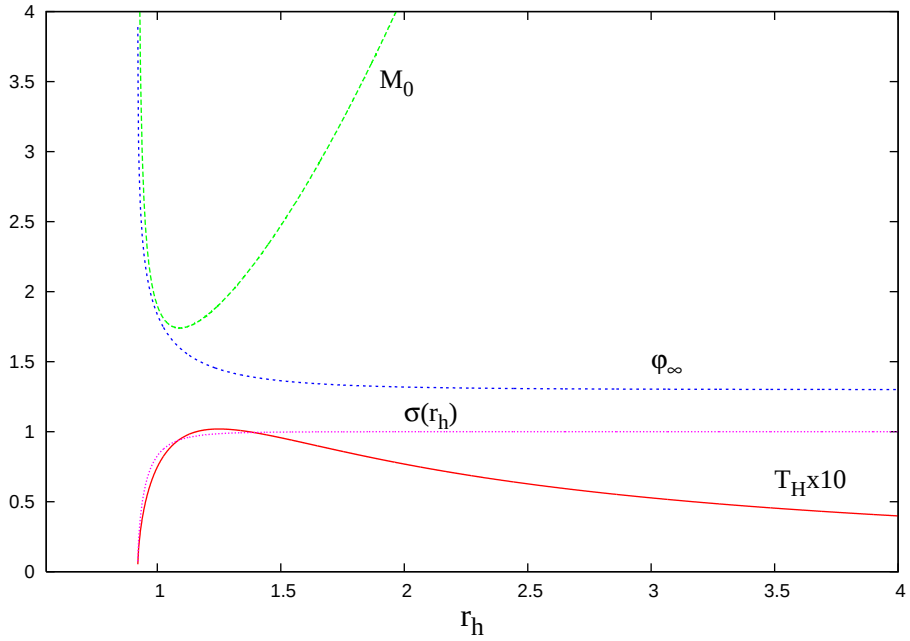

Figure 2: The relevant parameters are ploted as a function of the even horizon radius for $k = 1$ non-extremal black holes with $\Lambda = -0.001$, $\alpha' = 0.01$, $q_1 = 0.3$, $q_2 = 0.5$, $\alpha_1 = -\alpha_2 = 2$.

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of two typical solutions with different values of $\phi(r_h)$. The evolution of the solution data as a function of the event horizon radius is reported on figure 2. For small values of r_h the numerical analysis strongly suggests that an extremal black hole solution is approached for a critical non-zero value of the event horizon radius.

3.3 Configurations with a massive scalar field

The situation is different when allowing the scalar field to present a mass term. As discussed in the last years by various authors, the field equations (3.10) with $\alpha' = 0$ admits a variety of solutions. This includes also configurations where the asymptotic behaviour of the scalar field is assumed to be slower than that of a localized distribution of matter. By relaxing the standard asymptotic conditions for asymptotically AdS solutions, it is possible to preserve the original symmetries at infinity, while the conserved charges are modified by including matter field terms (see *e.g.* [36] and the references there).

It would be interesting to see how these features are affected by the presence of a GB term in the action. In this context, we start by discussing the issue of Breitenlohner-Freedman (BF) bound for a scalar field in the presence of a GB term in the Lagrangean. One can easily see that for $\alpha' > 0$, the mass bound increases according to $\mu^2_{BF} = -4/L^2_{\text{eff}}$. For example, the generic asymptotic behaviour of the scalar field $\phi(r)$ in the background (3.7) is

$$\phi(r) = \frac{\phi_1}{r^{\lambda_+}} + \frac{\phi_2}{r^{\lambda_-}} \tag{3.20}$$

where ϕ_1, ϕ_2 are constants and

$$\lambda_\pm = 2\left(1 \pm \sqrt{1 - \mu^2/\mu^2_{BF}}\right) \tag{3.21}$$

Here we assume $\mu^2 L^2_{\text{eff}} + 4 \geq 0$. Imposing that both the λ_- and λ_+ solutions be normalizable results in a supplementary conditions on the parameter $\mu^2 L^2_{\text{eff}} + 3 < 0$. For fields that saturate the BF bound, $\lambda_+ = \lambda_-$ and the solution is $\phi(r) = \phi_1/r^\lambda + \phi_2 \log r/r^\lambda$.

We shall restrict here to a scalar field ϕ that is tachyonic ($\mu^2 < 0$) and its mass is in the BF range, i.e. decaying at infinity according to (3.20) and not saturating the BF bound. We also consider solutions with a well defined Einstein gravity limit.[6]

In this case, the conserved charges are well defined and finite despite the fact that the scalar field falls off slower than usual. The analysis in the procedure of holographic renormalization is based on finding the most general asymptotic solution of the field equations [38] and so the fall off of the matter fields is not an input in the computation.[7] Considering now the question of action and total mass-energy of these solutions, one can see that for $\phi_2 \neq 0$, due to the back reaction of the scalar field, the boundary counterterms (2.9) are not enough to cancel all divergences in the on-shell action of the solutions.

[6] A detailed discussion on the asymptotic form of the metric and the implications for the no-hair theorem can be find in [37]. In our case, the only difference is that the solutions approach at the boundary an AdS space with a modified radius, L_{eff} given by (2.4).

[7] We thank Kostas Skenderis for discussions on holographic renormalization method for this case.

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Therefore one has to supplement I_{ct} with matter counterterms [39]. The counterterms (which include terms beyond the purely gravitational ones) are local and the action (and, also, the holographic stress energy tensor) has a contribution from the scalar fields.

As a result, the total action of the solutions is

$$I = \beta\left[\frac{V_k}{8\pi G}\left(\frac{3M_0}{2}+M_\phi\right)+M_{\rm casimir}-Q_1\Phi^1-Q_2\Phi^2\right]-\frac{1}{4G}(r_h^3+12k\alpha' r_h)V_k \quad (3.22)$$

where M_0 is the usual mass parameter, $M_\phi = \lambda_- \phi_1\phi_2/L^2_{\rm eff}$ (see, e.g., [37]), and the Casimir contribution is

$$M_{\rm casimir} = k^2\frac{3L^2_{\rm eff}}{64\pi G}V_k(3U-2) \quad (3.23)$$

Thus the mass contains a supplementary term due to the slower decay of the scalar field. This dependence is crucial for the stress tensor to satisfy the correct Ward identities and to lead to sensible results.

However, from the Gibbs-Duhem relation (2.29) one finds that the entropy of these solutions is still given by the relation (2.32).

4. Extremal solutions and attractor mechanism

The numerical extremal solutions can also be discussed by using similar methods as in previous section. However, the situation in this case is more involved. Similar to the non-extremal case, one may write an approximate form of these configurations near the event horizon — for static extremal black holes the near horizon geometry is $AdS_2\times S^3$. Let us first investigate the near horizon geometry of these black holes by using the entropy function formalism. For simplicity, we are again considering a theory with one scalar field and two U(1) (electric) gauge fields with the couplings given by (3.15). The general metric of $AdS_2\times S^3$ can be written as

$$ds^2 = v_1\left(-\rho^2 d\tau^2+\frac{1}{\rho^2}d\rho^2\right)+v_2 d\Omega_3^2 \quad (4.1)$$

The field strength ansatz is $F^A = e^A d\tau\wedge d\rho$. Thus, the entropy function $F(v_1,v_2,e^A,q_A,\phi_h)$ is similar with the one in section 2, except that we have now non-trivial couplings between scalars and the U(1) fields:

$$F(v_1,v_2,e^A,q_A,\phi_h) = 2\pi[q_A e^A - f(v_1,v_2,e^A,q_A,\phi_h)]\,, \quad (4.2)$$

$$f(v_1,v_2,e) = 2\pi^2\Big[-2v_2^{3/2}+6v_1\sqrt{v_2}+2\frac{v_2^{3/2}}{v_1}f_{AB}(\phi_h)e^Ae^B + v_1v_2^{3/2}\left(\frac{12}{L^2}\right)-24\alpha'\sqrt{v_2}\Big]\,. \quad (4.3)$$

By using the same trick as in section 2 we can compute the horizon radius and the value of the scalar at the horizon by solving the following equations

$$\alpha_1 e^{-\alpha_1\phi_h}q_1^2+\alpha_2 e^{-\alpha_2\phi_h}q_2^2=0,\quad e^{-\alpha_1\phi_h}q_1^2+e^{-\alpha_2\phi_h}q_2^2=3r_h^4\left(k+\frac{r_h^2}{L^2}\right)$$

in terms of q_1, q_2 and α_1, α_2.

While $\phi_h = \frac{1}{\alpha_2-\alpha_1}\log(-(\alpha_2 q_2^2)/(\alpha_1 q_2^1))$, the expression of $r_h(q_1, q_2, L)$ is very complicated and we do not present it here — note though that the relation between the horizon radius and the charges is similar with (2.43) where Q^2 is replaced by $q_1 q_2$. The horizon value of the scalar does not depend of the boundary value ϕ_∞ and so the near horizon geometry is universal. Consequently, the entropy of the extremal black hole does not depend of the boundary values of the scalar field. The behaviour of the scalar field is illustrated in figure 3 and the attractor mechanism is a direct consequence of the extremality condition.

At this point, it is worth trying to find a whole solution interpolating between the horizon and the boundary — the entropy function assumes the existence of such a solution but does not prove it.

Unlike in the non-extremal case, the value ϕ_h of the scalar field on the horizon and the event horizon radius r_h are not free parameters and so the horizon data contain two essential parameters. The leading terms in this expansion read

$$\begin{aligned} N(r) &= \frac{4(kL^2+3r_h^2)}{L^2(r_h^2+4\alpha' k)}(r-r_h)^2+\dots, \\ \sigma(r) &= \sigma_h + \frac{2p^2 r_h^3 \sigma_h \phi_1^2}{3(r_h^2+4\alpha' k)(2p-1)}(r-r_h)^{2p-1}+\dots, \\ \phi(r) &= \phi_h + \phi_1 (r-r_h)^p \ +\dots \end{aligned} \tag{4.4}$$

where

$$p = \frac{1}{2}\left(-1+\sqrt{1-3\alpha_1\alpha_2\frac{\left(k-\frac{2r_h^2}{L^2}\right)\left(1+\frac{4\alpha' k}{r_h^2}\right)}{k-\frac{3r_h^2}{L^2}}}\right)$$

Similar to the nonextremal case, we evaluate the initial conditions (4.4) at $r = r_h + 10^{-4}$ for global tolerance 10^{-14}, integrating towards $r \to \infty$. The large r expansion of the extremal solutions is still given by the expression (3.18). Similar to the non-extremal case, the value of σ_h in the horizon data is not relevant in numerics and the only parameter is ϕ_1. Again, we did not notice the existence of local extrema of the functions $N(r)$, $\sigma(r)$, $\phi(r)$.

In figure 4 we have ploted the profiles of a typical extremal black hole with non-zero GB term together with the corresponding solution with $\alpha' = 0$. One can see that a non-zero α' leads to a deformation of all metric functions at all scales, which holds also for the dilaton ϕ.

It is also possible to write a simple generalization of the Bertotti-Robinson solution (2.37), with the same line element and the matter fields scalar field

$$\phi_0 = \frac{1}{\alpha_2-\alpha_1}\log\left(-\frac{\alpha_2 q_2^2}{\alpha_1 q_2^1}\right), \quad W^1 = e^{-\alpha_1\phi_0} q_1 r, \quad W^2 = e^{-\alpha_2\phi_0} q_2 r, \tag{4.5}$$

the size of Σ_3^2 being fixed by the cosmological constant and the parameters q_1, q_2 as solution of the equation

$$\frac{3k}{R_\Sigma^2}+\frac{6}{L^2} = e^{-\alpha_1\phi_0} q_1^2 + e^{-\alpha_2\phi_0} q_2^2. \tag{4.6}$$

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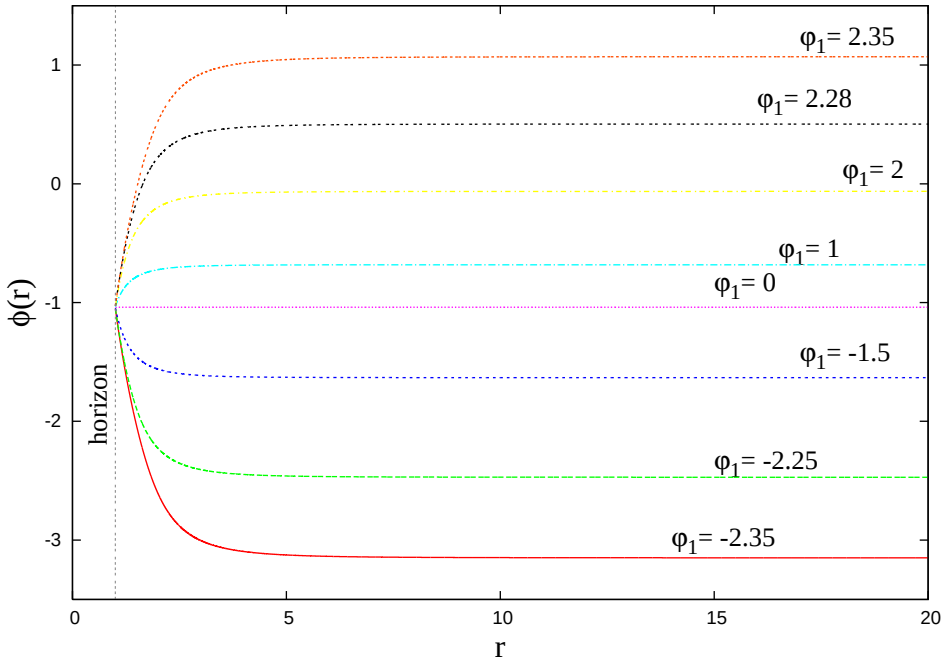

Figure 3: The attractor behaviour is shown for $k = 1$ extremal black holes with $\Lambda = -1$, $\alpha' = 0.1$, $q_1 = 4$, $q_2 = 0.5$, $\alpha_1 = -\alpha_2 = 1/2$.

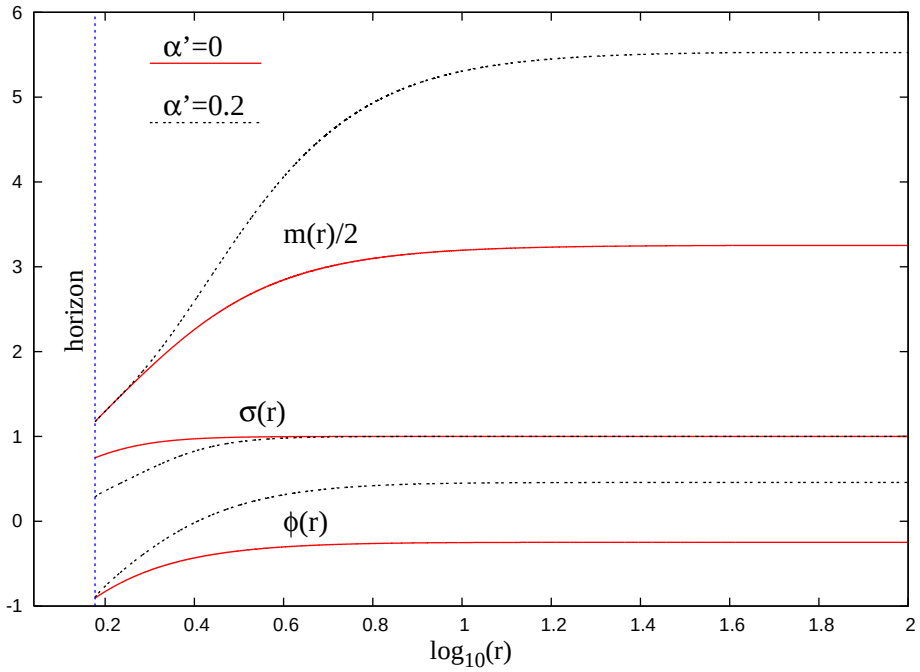

Figure 4: The profiles of the functions $m(r)$, $\sigma(r)$ and $\phi(r)$ are shown for a typical $\alpha' = 0.2$, black hole with $k = 1$, $\alpha_1 = -\alpha_2 = 1/2, \phi_1 = 1, q_1 = 7.1, q_2 = 1.15$. For comparison, we included also the profiles of the corresponding solution in Einstein gravity ($\alpha' = 0$).

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5. Discussion

In this paper, we have investigated the construction of black hole solutions in higher derivative AdS gravity. This is a self contained paper and we hope that our unified treatment of (non-)extremal AdS black hole solutions with GB term is useful to the reader.

In the presence of higher derivative terms the area law is modified. Wald *derived* a new formula for the themodynamical entropy such that the first law of black hole mechanics remains valid. In this paper, we have taken a slightly different route to studying thermodynamical properties of these black holes — we have checked the first law in several concrete examples.

The main tool that we have used in the non-extremal case is the counterterm method. We have explicitly constructed the counterterms that regularize the action and the stress tensor and applied the holographic renormalization method in several concrete examples[8] — we found perfect agreement with the results obtained by Wald formalism. The method, as developed in [38], constructs unambigously the counterterms that render the action finite on any solution of the field equations. In other words, the method does not proceed with postulating a set of counterterms and then checking that these work for a class of solutions. Rather one first obtains the behaviour of the on-shell action on the most general solution and then from here one finds the set of counterterms. It would be interesting to understand better how the counterterms we proposed fit in this general framework.

AdS spacetime is geodesically complete, but the light cones flare out in such a way that particles can exit from the space — and also information can come into the space — within a finite time. Thus, AdS is not a globally hyperbolic spacetime and so the boundary conditions should play an important role. Indeed, within the AdS/CFT duality, various deformations of the AdS boundary conditions are interpreted as dual to deformations of the CFT. If such a CFT exists the theory lies in the landscape of string theory and the bulk theory is manifestly consistent as an effective theory, otherwise the theory is part of the swampland [25].

A 'ground state' is defined as a state that extremizes the Hamiltonian over the class of vacuum states which all have a given boundary topology. It is well known that by using different foliations of AdS space one can describe boundaries that have different topologies affording the study of CFT on different backgrounds. The diffeomorphisms in the bulk are equivalent with the conformal transformations in the boundary, and different boundary topologies are related by *singular* conformal transformations. Therefore, for AdS gravity, the correct variational problem requires keeping fixed a conformal structure rather than a boundary metric. The variational problem for two derivative AdS gravity was discussed in detail in [41]. This paper also analyzes Wald's method for asymptotically locally AdS spacetimes, shows in generality that the results of this method agree with the holographic results, and establishes the 1st law of thermodynamics.

Our results fit in this general framework. For black holes with a boundary topology of $R \times S^3(H^3)$ we found additional Casimir-type contributions to the energy. That is in

[8]We also have checked this method for other solutions with non-trivial boundary topology [40], but we hope to present these results elsewhere.

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accord with the expectations from quantum field theory in curved space: for the Casimir effect, the global structure is reflected nontrivially in the ground state of the quantum field.

We have also constructed extremal solutions and investigated their properties. Wald formalism was extended by Sen to extremal black holes. The advantage of this method is that the higher derivatives terms can be incorporated easily, but the method can not be used to determine the properties of the solution away from the horizon. However, in section 4, we have constructed numerical solutions that interpolate between the horizon and the boundary. Thus, we were able to safely apply the entropy function formalism to study their properties.[9]

Unlike the non-extremal case where the near horizon geometry (and the entropy) depends on the boundary values of the moduli, in the extremal case, the near horizon geometry is universal and is determined by only the charge parameters. We have also constructed numerical solutions for which the near horizon geometry is $AdS_2 \times H^3(T^3)$, though we do not present the details here. We have found that, in all these cases, the scalar fields are attracted to fixed values at the horizon. This does not come as a surprise since it is known that the 'long throat' of AdS_2 is at the basis of attractor mechanism.

A detailed analysis of the attractor mechanism and interpretations within the AdS/CFT duality can be find in [15].[10] After embedding in string theory, the moduli flow becomes a holographic renormalization group (RG) flow. The idea that the IR endpoint of a QFT RG flow does not depend upon UV details becomes in the holographic context the statement that the bulk solution in the near horizon limit does not depend upon the details at the boundary (asymptotic values of the moduli). That is a holographic interpretation of attractor mechanism [15].

When the scalars potential is not a constant, a general analysis of the attractor mechanism is difficult. First of all, if the boundary values of the moduli are fixed to a minimum of the potential it is not clear how 'to fly' to IR horizon where the moduli may get different values depending of charges (the existence of extremal solutions in this case is problematic). However, if the potential has flat directions it may be possible to perturb along these directions. Therefore, a discussion of the attractor mechanism for a non constant scalars potential should be made case by case.

Although the focus of this paper has been on solutions in higher derivative AdS gravity, it will be interesting to develop a similar technique for asymptotically flat solutions. In particular, it will be interesting to find a boundary stress tensor analogous to the one for two derivative gravity [44].

Acknowledgments

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[9] Entropy function formalism was applied to black holes in AdS space in [15, 42].

[10] In this paper we are interested in AdS black holes, for more details on attractor mechanism in flat space one can consult the nice reviews [43] and references therein.

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A. Wald formalism for Gauss-Bonnet action

The action in presence of generalised Gauss-Bonnet term is of the form:

$$I = \int d^5x\sqrt{-g}\left[\frac{R}{16\pi G} - 2\Lambda - \frac{F_{\mu\nu}F^{\mu\nu}}{16\pi G} + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu} + \gamma R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right] \tag{A.1}$$

Using Wald formalism, the entropy in presence of this term is given by

$$S = \frac{1}{4G}\int_{\mathcal{H}} d^3x\sqrt{h}\left[1 + 2K_5\alpha R + K_5\beta(R - h^{ij}R_{ij}) + 2K_5\gamma(R - 2h^{ij}R_{ij} + h^{ij}h^{kl}R_{ikjl})\right] \tag{A.2}$$

where h is the induced metric on the boundary and $K_5 = 16\pi G$.

Here, we present a detailed proof of this formula.

General Wald formula for the entropy for any Lagrangean L is

$$S = -2\pi\int_{\mathcal{H}} d^3x\sqrt{h}\frac{\partial L}{\partial R_{abcd}}\epsilon_{ab}\epsilon_{cd} \tag{A.3}$$

where $\mathcal{H}$ is the bifurcate horizon and $\epsilon_{\mu\nu}$ is the binormal to the bifurcation surface, normalized such that $\epsilon_{\mu\nu}\epsilon^{\mu\nu} = -2$.We can take

$$\epsilon_{\mu\nu} = \xi_\mu\eta_\nu - \xi_\nu\eta_\mu, \tag{A.4}$$

where ξ and η are the null vectors normal to the bifurcate Killing horizon, with $\xi.\eta = 1$. We will take

$$\xi = \frac{\partial}{\partial t} \tag{A.5}$$

which is null at the bifurcate horizon. Then η can be

$$\eta = -\frac{1}{g_{tt}}\frac{\partial}{\partial t} - \frac{\partial}{\partial r} \tag{A.6}$$

Now with all these definitions, we can proceed to compute the entropy using Wald formalism. We can write the Einstein-Hilbert term using its symmetries as

$$R = \frac{1}{2}(g^{ac}g^{bd} - g^{ad}g^{bc})R_{abcd} \tag{A.7}$$

So the leading piece in entropy is

$$S_0 = -2\pi\int d^x\sqrt{h}\frac{1}{16\pi G}\frac{\partial R}{\partial R_{abcd}}\epsilon_{ab}\epsilon_{cd} = \frac{A}{4\pi} \tag{A.8}$$

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R^2 part:

$$\frac{\partial R^2}{\partial Rabcd}\epsilon_{ab}\epsilon_{cd} = -4R \tag{A.9}$$

So, the contribution to the Entropy is,

$$S_1 = \frac{1}{4G}\int d^3x\sqrt{h}2K_5\alpha R \tag{A.10}$$

$R^{ij}R_{ij}$ part:

$$\frac{\partial(R^{ij}R_{ij})}{\partial Rabcd}\epsilon_{ab}\epsilon_{cd} = 2R^{ij}g^{kl}\delta^a_k\delta^b_i\delta^c_l\delta^d_j\epsilon_{ab}\epsilon_{cd} = 2R^{bd}\epsilon_{ab}\epsilon^a_d = -2R^{bd}(\xi_b\eta_d + \xi_d\eta_b), \tag{A.11}$$

where we have used the definition of binornal. Now, using the following relation between the induced metric and the original metric

$$h_{bd} = g_{bd} - (\xi_b\eta_d + \xi_d\eta_b) \tag{A.12}$$

we get that the contribution to the entropy is

$$S_2 = \frac{1}{4G}\int d^3x\sqrt{h}K_5\beta(R - h^{ab}R_{ab}) \tag{A.13}$$

$R^{ijkl}R_{ijkl}$ part:

$$\begin{aligned}\frac{\partial(R^{ijkl}R_{ijkl})}{\partial Rabcd}\epsilon_{ab}\epsilon_{cd} &= 2R^{ijkl}\delta^a_i\delta^b j\delta^c_k\delta^d_l \\ &= R^{abcd}(\xi_a\eta_b - \xi_b\eta_a)(\xi_c\eta_d - \xi_d\eta_c) \\ &= -2R^{abcd}(g_{ac} - h_{ac})(g_{bd} - h_{bd}) \\ &= -2(R - 2h^{bd}R_{bd} + h^{ac}h^{bd}R_{abcd}).\end{aligned} \tag{A.14}$$

So the contribution to the entropy is

$$S_3 = \frac{1}{4G}\int d^3x\sqrt{h}2K_5\gamma(R - 2h^{bd}R_{bd} + h^{ac}h^{bd}R_{abcd}) \tag{A.15}$$

Thus we get the net entropy due to the presence of the GB term in the action as $S = S_0 + S_1 + S_2 + S_3$ — we used this expression in section 2.

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